Physics of Sustainable Energy

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What math do you need to know?

- Square roots
- Scientific ("exponential") notation for numbers
- Unit conversions

And, even more importantly,

How to do *rough estimates*,
and avoid unnecessary precision.

In our calculations, we only want to get a *roughly right* answer.
20% accuracy is usually good enough. Sometimes we'll only need the answer to be correct to within a factor of 2.

E.g.,
Q: What's the population of China?
A: about 1.4 billion NOT "1,445,690,926"

Scientific ("exponential") notation

$$
\begin{aligned}
3{,}000{,}000 &= 3 \times 10^6 \\
\ldots & \\
3000 &= 3 \times 10^3 \\
300 &= 3 \times 10^2 \\
30 &= 3 \times 10^1 \qquad 10^1 = 10 \\
3 &= 3 \times 10^0 \qquad 10^0 = 1 \\
0.3 &= 3 \times 10^{-1} \quad 10^{-1} = 0.1 \\
0.03 &= 3 \times 10^{-2} \\
0.003 &= 3 \times 10^{-3} \\
\ldots & \\
3 \text{ millionths} &= 3 \times 10^{-6}
\end{aligned}
$$

Exercise

What fraction of the population of the USA lives in Missouri?

(A) 2%   (B) 1%   (C) 20%   (D) 10%

Metric prefixes

G	giga	billion	10^9	$1\,\mathrm{GW} = 10^9\,\mathrm{W}$ 1 giga-Watt = a billion Watts
M	mega	million	$1{,}000{,}000 = 10^6$	$1\,\mathrm{MT} = 10^6\,\mathrm{T}$ 1 mega-Tonne = a million Tonnes
k	kilo	thousand	$1{,}000 = 10^3$	$1\,\mathrm{kg} = 1000\ \mathrm{g}$
c	centi	hundredth	$0.01 = \frac{1}{100} = 10^{-2}$	$1\,\mathrm{cm} = 0.01\,\mathrm{m}$
m	milli	thousandth	$0.001 = \frac{1}{1000} = 10^{-3}$	$1\,\mathrm{mm} = 0.001\,\mathrm{m}$
μ	micro	millionth	$\frac{1}{1{,}000{,}000} = 10^{-6}$	$1\,\mu\mathrm{m} = 10^{-6}\,\mathrm{m}$
n	nano	billionth	10^{-9}	$1\,\mathrm{nm} = 10^{-9}\,\mathrm{m}$

Unit conversions

See course videos:

- Unit Conversions 1: basic technique
- Unit Conversions 2: multiple units

Exercise

One yard is 3 feet. What is one square yard, expressed in square feet?
(A) $1\,\mathrm{ft}^2$ (B) $3\,\mathrm{ft}^2$ (C) $9\,\mathrm{ft}^2$ (D) $0.33\,\mathrm{ft}^2$

The area of Missouri is 70,000 square miles. What is that in square kilometers?
(A) $70,000\,\mathrm{km}^2$ (B) $180,000\,\mathrm{km}^2$
(C) $122,000\,\mathrm{km}^2$ (D) $44,000\,\mathrm{km}^2$

Unit conversion toolkit, part 1

Start building your unit conversion toolkit:

Length:	$1\,\text{mi} = 1.6\,\text{km}$	$1\,\text{inch} = 2.5\,\text{cm}$	$1\,\text{foot} = 30\,\text{cm}$	$\ldots$
Volume:	$1\,\text{liter} = 10^3\,\text{cm}^3$ $1\,\text{ml} = 1\,\text{cm}^3$	$1\,\text{Gal} = 4\,\text{quarts}$ $1\,\text{quart} = 4\,\text{cups}$	$1\,\text{Gal} = 3.8\,\text{liters}$ $\approx 4\,\text{liters}$	

Exercise

What is 1 cup in milliliters?

(A) 50 ml (B) 380 ml (C) 250 ml (D) 120 ml

Exercise

How many liters are there in a cubic meter?

(A) 100 (B) 1000 (C) 10,000 (D) 1 million

Unit conversion toolkit, part 2

Mass: $1\,\text{lb} = 16\,\text{oz}$	$1\,\text{kg} = 2.2\,\text{lb}$ $\approx 2\,\text{lb}$	$1\,\text{ton} \approx 1000\,\text{kg}$	...

Exercise

Roughly how many pounds in a ton?

(A) 100 (B) 200 (C) 1000 (D) 2000

Density

Density of water is $1\,\frac{\text{kg}}{\text{liter}} = 1\,\frac{\text{g}}{\text{ml}} = 1\,\frac{\text{g}}{\text{cm}^3}$

Exercise

What is the mass of a cubic meter of water?

(A) 1 kg (B) 100 kg (C) 1 ton (D) 100 ton

Energy as "Money"

Energy is like Money.

The amount of energy you have available determines what you can get done.

Energy and Money are both *conserved*: they can be transferred between different places and take different forms, but the total amount is unchanged.

Except that governments can "print money".

- How do we obtain energy from our environment?
  Coal, Oil, Sunlight ...
- How do we store energy for future use?
  Gasoline, Batteries, Food ...
- How do we transport energy?
  Electric power lines, Oil tankers ...
- How do we use energy to get things done?
  Engines, Motors, Muscles, Light bulbs, Speakers...

Energy Topics

- Forms of energy
  Kinetic, Chemical, Nuclear, Electrical, Thermal
- Quantifying amounts of energy
  Units: Calories, kiloWatt-hours, etc
- Storing energy
  Batteries, Food, Fuels, Uranium... Which is "best"?
- **Application:** comparison of electric cars, hybrid cars, and gasoline cars
- Cost of energy
  Coal, Gas, Oil, Electricity, etc
- Power: rate of flow of energy

Forms of energy

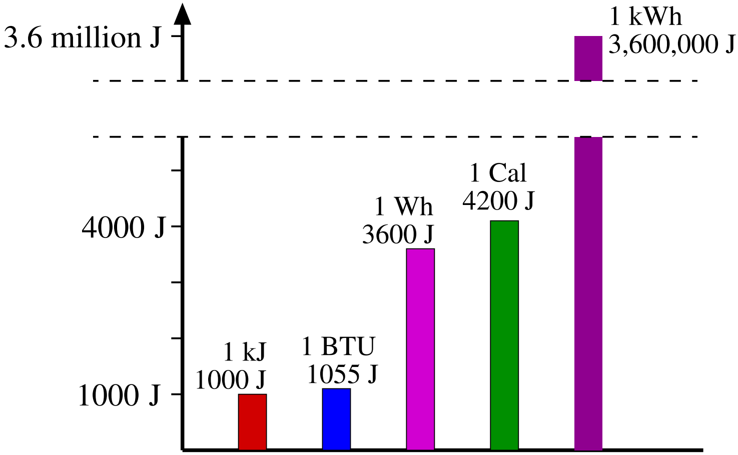
Energy is taken in or given out during a change of state

- Energy that a car has when it is driving at 50 mi/hr
  (moving → stationary) (Kinetic Energy)
- Energy stored in TNT, released when it explodes
  (TNT → hot gas of Nitrogen, water, etc) (Chemical Energy)
- Energy stored in a gram of butter, released when it is burned
  (fat + oxygen → water and CO_2) (Chemical Energy)
- Energy of a brick at the top of a building, released when it falls
  (high up → low down) (Gravitational Potential Energy)
- Energy stored in a uranium-235 nucleus, released when it decays
  (Uranium → two smaller nuclei) (Nuclear Energy)
- Energy stored in wires that are carrying an electric current
  (current flowing → no current flowing) (Electrical Energy)
- Energy stored as heat, released when hot object cools
  (hotter → cooler) (Thermal Energy)

Energy units

Joule (J)		Physicists' energy unit; a small amount of energy; energy needed to heat $\sim$10 drops of water by 1 C energy needed to raise 1 kg by about 10 cm
kiloJoule (kJ)	$=1000$ J	
Calorie	$=4200$ J	Typical unit of energy in food heats 1 kg of water by 1 C
BTU	$=1055$ J	British Thermal Unit heats 1 lb of water by 1 F
quad	$=10^{18}$ J	Quadrillion BTU$=10^{15}$ BTU used for energy of whole countries or industries E.g. total U.S. energy use ≈ 100 quads/year
Watt-hour (Wh)	$=3600$ J	energy used by a 1 W appliance running for 1 hr typical laptop battery stores about 60 Wh
kiloWatt-hour (kWh)	$=3.6\times10^6$ J	energy used by a 1 kW appliance running for 1 hr Electricity from a utility typically costs about 10 ¢/kWh

Energy units: visual comparison



Unit conversion toolkit, part 3

Add Energy conversions to the toolkit:

Length:	$1\,\text{mi} = 1.6\,\text{km}$	$1\,\text{inch} = 2.5\,\text{cm}$	$1\,\text{foot} = 30\,\text{cm}$	...
Volume:	$1\,\text{liter} = 10^3\,\text{cm}^3$ $1\,\text{ml} = 1\,\text{cm}^3$	$1\,\text{Gal} = 4\,\text{quarts}$ $1\,\text{quart} = 4\,\text{cups}$	$1\,\text{Gal} = 3.8\,\text{liters}$ $\approx 4\,\text{liters}$	...
Mass:	$1\,\text{lb} = 16\,\text{oz}$	$1\,\text{kg} = 2.2\,\text{lb}$ $\approx 2\,\text{lb}$	$1\,\text{ton} \approx 1000\,\text{kg}$	...
Energy:	$1\,\text{Cal} = 4200\,\text{J}$ $1\,\text{BTU} = 1055\,\text{J}$	$1\,\text{Wh} = 3600\,\text{J}$ ...	$1\,\text{kWh} = 3.6 \times 10^6\,\text{J}$	

Exercise

What is 1 kiloWatt-hour in Calories?

(A) 860 Cal (B) 4200 Cal (C) 3600 Cal (D) 3.6 Cal

Energy Storage: requirements

We have two requirements which sometimes conflict:

- Accessibility: energy can easily be extracted from “container” and used.
- High energy density: lots of energy stored per gram of “container”.

Energy storage table

Storage medium	$\frac{\text{Calories}}{\text{gram}}$	$\frac{\text{Joules}}{\text{gram}}$	change of state to extract energy
Flywheel	0.01–0.1	50–500	moving $\rightarrow$ stationary
Battery lead-acid, car	0.03	140	charged $\rightarrow$ discharged
Battery (flow)	0.02	100	charged $\rightarrow$ discharged
Battery (lithium-ion, computer, elec car)	0.05 − 0.2 typical: 0.1	200 − 800 typical: 400	charged $\rightarrow$ discharged
Battery (alkaline, AA, AAA, flashlight etc)	0.15	600	charged $\rightarrow$ discharged
Tri-nitro toluene (TNT explosive)	1	4200	TNT $\rightarrow$ N_2, H_2, CO
Carbohydrates, protein	4	17,000	$+ O_2 \rightarrow CO_2 + H_2O$
Coal	6	27,000	$+ O_2 \rightarrow CO_2$
Alcohol (ethanol, methanol)	6	27,000	$+ O_2 \rightarrow CO_2 + H_2O$
Butter	7	29,000	$+ O_2 \rightarrow CO_2 + H_2O$
Fat	9	38,000	$+ O_2 \rightarrow CO_2 + H_2O$
Gasoline	10	42,000	$+ O_2 \rightarrow CO_2 + H_2O$
Natural gas (methane = CH_4)	13	54,000	$+ O_2 \rightarrow CO_2 + H_2O$
Hydrogen (gas or liquid)	26	110,000	$+ O_2 \rightarrow H_2O$
Uranium-235	20 million	83 billion	$^{235}U \rightarrow$ 2 smaller nuclei

Electric car vs Gasoline car, generalities

What do we compare?
(1) Range
(2) Cost per mile

Relevant differences:

Energy Storage:
Gasoline cars use gasoline
Electric cars use batteries.
Gasoline has 100 times higher energy/gram than batteries!

Energy Usage:
Gasoline engines are $\sim$ 20% efficient,
electric motors are $\sim$ 80% efficient

Electric motor is 4 times more efficient than gasoline engine.

Electric car vs Gasoline car, specifics

(1) Range: How far can we go on:

- *Gasoline car:* Full tank of gasoline
- *Electric car with Lithium-ion batteries:* Fully charged battery pack

(2) Cost per mile:

- *Gasoline car:* $\dfrac{\text{Cost}}{\text{kWh of energy in fuel}} \times \dfrac{\text{kWh of energy in fuel}}{\text{Miles traveled}}$
- *Electric car:* $\dfrac{\text{Cost}}{\text{kWh of energy in battery}} \times \dfrac{\text{kWh of energy in battery}}{\text{Miles traveled}}$

Other issues:

- Cost of vehicle, including battery pack
- Upkeep costs

Exercises

How far can an electric car travel if it has a fully charged Li-ion battery weighing 100 lb ?

(A) 1 mile (B) 4 miles (C) 12 miles (D) 30 miles

How many kWh of energy are there in a gallon of gasoline?

(A) 35 kWh (B) 15 kWh (C) 3 kWh (D) 7 kWh

What is the cost of each kWh of energy in gasoline?

(A) 3¢ (B) 10¢ (C) 30¢ (D) $ 1