Decay of superfluid vortices in CFL quark matter

Prof. Mark Alford
Washington University in St. Louis

Alford, Mallavarapu, Vachaspati, Windisch
arXiv:1601.04656 (Phys Rev C)

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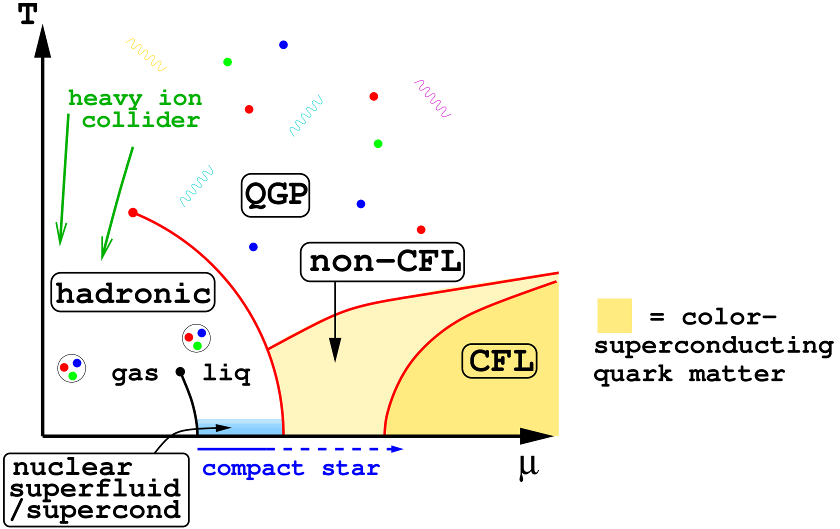
Outline

- Color-flavor locked quark matter: a superfluid.
- The instability of CFL superfluid vortices:
  Mystery 1 Why are they not stable?
  Mystery 2 Are they Metastable or Unstable?
- Answer 1: Semi-superfluid flux tubes are the lower-energy alternative to vortices.
- Answer 2: It depends on the couplings. We numerically mapped the metastability boundary.
- Bonus: the unstable mode, analytically understood
- Conclusions

Schematic QCD phase diagram



M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, arXiv:0709.4635 (RMP review)

A. Schmitt, arXiv:1001.3294 (Springer Lecture Notes)

Color superconducting phases

Attractive QCD interaction $\Rightarrow$ Cooper pairing of quarks.

Quark Cooper pair: $\langle q^{\alpha}_{a\xi} q^{\beta}_{b\zeta} \rangle$

color $\alpha, \beta = r, g, b$
flavor $a, b = u, d, s$
spin $\xi, \zeta = \uparrow, \downarrow$

Each possible BCS pairing pattern P is an 18×18 color-flavor-spin matrix

$$\langle q^{\alpha}_{a\xi} q^{\beta}_{b\zeta} \rangle_{1PI} = \Delta_P \, P^{\alpha\beta}_{ab\,\xi\zeta}$$

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We expect pairing between *different flavors*.

The attractive channel is:

	space symmetric	[s-wave pairing]
	color antisymmetric	[most attractive]
	spin antisymmetric	[isotropic]
$\Rightarrow$	flavor antisymmetric	

We will assume the most symmetric case, where all three flavors are massless.

Color-flavor-locked quark matter

Equal number of colors and flavors gives a special pairing pattern
(Alford, Rajagopal, Wilczek, hep-ph/9804403)

$$\langle q_a^\alpha q_b^\beta \rangle \sim \delta_a^\alpha \delta_b^\beta - \delta_b^\alpha \delta_a^\beta = \epsilon^{\alpha\beta n} \epsilon_{abn}$$

color α, β
flavor a, b

This is invariant under equal and opposite rotations of color and (vector) flavor

$$SU(3)_{\text{color}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{C+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

Additional factors of $\mathbb{Z}_3$ not shown

- Breaks baryon number $\Rightarrow$ superfluid $\Rightarrow$ **vortices**
- Breaks chiral symmetry, but *not* by a $\langle \bar{q} q \rangle$ condensate.
- Is there a phase transition between the low and high density phases: ("quark-hadron continuity") ?

Mysteries of superfluid vortices in CFL

CFL quark matter is a superfluid so angular momentum is carried by vortices where the phase of the quark condensate (all components) circulates around the core.

At large r,
$\langle qq \rangle \sim e^{i\theta}$

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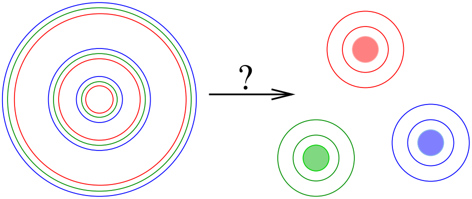
Mystery 1:

These vortices are **not stable**!
A configuration of 3 well-separated "semisuperfluid flux tubes" has lower energy than a vortex.



Balachandran, Digal, Matsuura, hep-ph/0509276

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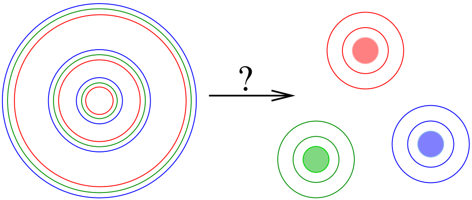
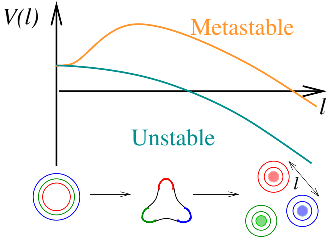


Mystery 2:

Are the vortices:
Metastable: there is an energy barrier
Unstable: they spontaneously fall apart



Effective theory of CFL condensate

Express the condensate as a scalar field Φ.

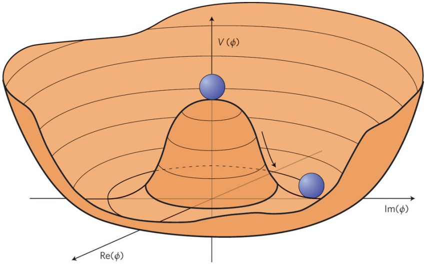
$$\Phi^a_\alpha = \epsilon_{\alpha\beta\gamma}\epsilon^{abc}\langle q^\beta_b q^\gamma_c \rangle$$

Φ is a 3×3 color-flavor matrix with baryon number $\frac{2}{3}$.
Φ couples to gluons. We neglect electromagnetism.

$$\begin{aligned} \mathcal{H} &= \tfrac{1}{4} F_{ij}F^{ij} + D_i\Phi^\dagger D^i\Phi + U(\Phi) \\ U(\Phi) &= m^2 \mathrm{Tr}[\Phi^\dagger\Phi] + \lambda_1(\mathrm{Tr}[\Phi^\dagger\Phi])^2 + \lambda_2 \mathrm{Tr}[(\Phi^\dagger\Phi)^2] \end{aligned}$$

If $m^2 < 0$, the ground state is

$$\langle\Phi\rangle = \begin{array}{c} \\ \color{red}{\mathrm{r}} \\ \color{green}{\mathrm{g}} \\ \color{blue}{\mathrm{b}} \end{array} \begin{array}{c} \mathrm{u}\quad \mathrm{d}\quad \mathrm{s} \\ \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \end{array} \bar{\phi}$$



The CFL superfluid vortex

The VEV of Φ breaks baryon number $\Rightarrow$ superfluidity.
The superfluid vortex is

$$A_i = 0 \ , \quad \Phi^{(\mathrm{sf})}{}^a_\alpha = \bar\phi\, \delta^a_\alpha \times e^{i\theta} \beta(r)$$

(It depends only on m^2 and $\lambda \equiv 3\lambda_1 + \lambda_2$.)

$$\Phi^{(\mathrm{sf})}{}^a_\alpha = \begin{array}{c} \\ \mathrm{r} \\ \mathrm{g} \\ \mathrm{b} \end{array} \begin{array}{c} \mathrm{u} \quad \mathrm{d} \quad \mathrm{s} \\ \begin{pmatrix} e^{i\theta} & & \\ & e^{i\theta} & \\ & & e^{i\theta} \end{pmatrix} \end{array} \bar\phi\, \beta(r)$$

This looks like a topologically stable configuration consisting of three superimposed global vortices, but it is not stable!
(Balachandran, Digal, Matsuura, hep-ph/0509276; Eto, Nitta, arXiv:0907.1278)

Mystery 1: How could there be a lower energy configuration?

$U(1)$: Global vortex vs Local flux tube

Vortex (global)	Flux tube (local)
e.g. vortex in a sf, like liq He	e.g. flux tube in type-II supercond.
Energy $\sim \log($Volume$)$	Energy is finite
Strong long range repulsion	Could attract or repel

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Far from core, $U(\phi) \to 0$

$$\phi(r,\theta) = \bar{\phi}\, e^{in\theta}$$

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$$\varepsilon \propto |\vec{\nabla}\phi|^2 = n^2 \bar{\phi}^2 / r^2 \qquad\qquad \varepsilon \propto |\vec{D}\phi|^2 = |\vec{\nabla}\phi - ig\vec{A}\phi|^2 = 0$$

$$E_{\text{vortex}} \sim E_{\text{core}} + n^2 \bar{\phi}^2 \ln\left(\frac{R_{\text{box}}}{R_{\text{core}}}\right) \qquad\qquad E_{\text{flux tube}} \sim E_{\text{core}}$$

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Vortex:

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Two $n=1$ vortices have half the energy of one $n=2$ vortex

Flux tube:

$$\phi(r,\theta) = \bar{\phi}\, e^{in\theta}$$

$$A_\theta = -\frac{n}{gr}$$

$$\varepsilon \propto |\vec{D}\phi|^2 = |\vec{\nabla}\phi - ig\vec{A}\phi|^2 = 0$$

$$E_{\text{flux tube}} \sim E_{\text{core}}$$

Two $n=1$ flux tubes could have more or less energy than one $n=2$ flux tube

Global vs local for $SU(3)$

CFL superfluid vortex is like three $n=1$ U(1) global vortices, "red up", "green down", "blue strange",

$$\Phi^{(\mathrm{sf})} \approx \bar{\phi} \begin{pmatrix} e^{i\theta} & & \\ & e^{i\theta} & \\ & & e^{i\theta} \end{pmatrix} \quad A_\theta^{(\mathrm{sf})} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Energy density $\quad \varepsilon \sim 3 \times 1^2 \times \bar{\phi}^2 / r^2 = 3\dfrac{\bar{\phi}^2}{r^2}$

Gauge fields can cancel out the gradient energy from the winding of the scalar field at large r.

Could we use color gauge fields to lower the energy of the CFL superfluid vortex?

There is no $U(1)_B$ gauge field, so we can't cancel *all* the gradient energy, but still. . .

The "semi-superfluid" flux tube

$$\Phi^{(\mathrm{ssf})} \approx \bar{\phi} \begin{pmatrix} e^{i\frac{\theta}{3}} & & \\ & e^{i\frac{\theta}{3}} & \\ & & e^{i\frac{\theta}{3}} \end{pmatrix} \times \begin{pmatrix} e^{-i\frac{\theta}{3}} & & \\ & e^{-i\frac{\theta}{3}} & \\ & & e^{i\frac{2\theta}{3}} \end{pmatrix} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\theta} \end{pmatrix}$$

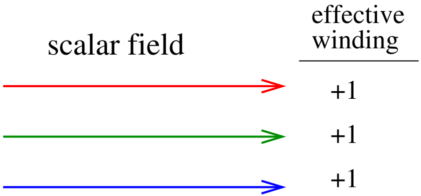
$$A_\theta^{(\mathrm{ssf})} = \frac{1}{g\,r}\frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Global vortex, $n = \frac{1}{3}$ — Local vortex

Far from core, $\varepsilon \sim 3 \times (\frac{1}{3})^2 \times \frac{\bar{\phi}^2}{r^2} = \frac{1}{3}\frac{\bar{\phi}^2}{r^2}$ vs $3\frac{\bar{\phi}^2}{r^2}$ for sf vortex

Using color flux to cancel $U(1)$ winding

Superfluid vortex

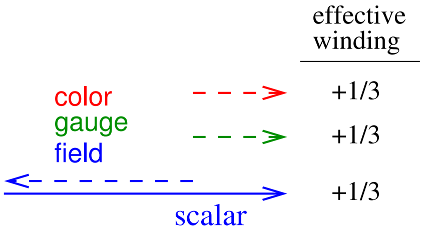


Total winding (ang mom): $+3$

Energy density:

$|\vec{\nabla}\Phi|^2 \sim 3 \times (+1)^3 = 3$

Semi-sf flux tube



Total winding (ang mom): $+1$

Energy density

$|\vec{D}\Phi|^2 \sim 3 \times (1/3)^3 = 1/3$

Mystery 1 solved

Mystery 1:

Why do three well-separated semi-superfluid flux tubes have lower energy than a vortex?

Answer 1 :

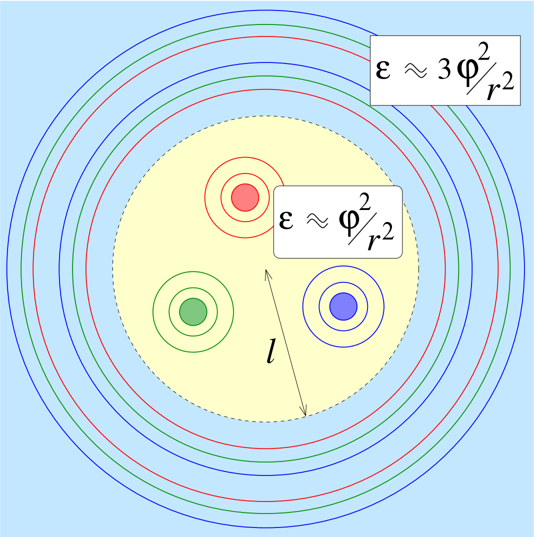
The semi-superfluid flux tubes use color gauge fields to cancel the gradient energy of *part* of the winding.

one sf vortex	one semi-sf flux tube
$\varepsilon \sim 3\bar{\phi}^2/r^2$	$\varepsilon \sim \frac{1}{3}\bar{\phi}^2/r^2$

We need 3 semi-sf flux tubes to carry the same ang mom as one sf vortex, but that still has lower energy then the vortex

Long range repulsion

The semisuperfluid flux tubes have a strong *long-range* repulsion:



$r \lesssim l$ low energy density

$$\varepsilon \sim 3 \times \tfrac{1}{3}\bar{\phi}^2/r^2 = \phi^2/r^2$$

$r \gtrsim l$ high energy density

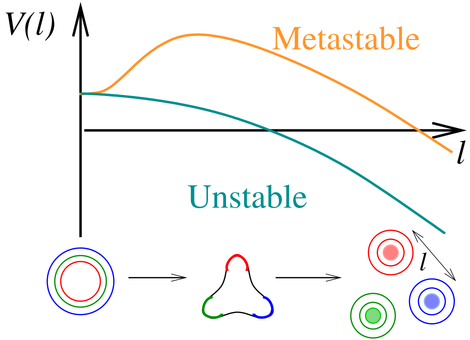
$$\varepsilon \sim 3\,\bar{\phi}^2/r^2$$

So as l rises, more of space contains low energy density.

$$V(l) \sim \bar{\phi}^2 \int_0^l r\,dr \frac{(1-3)}{r^2}$$

$$\sim \text{const} - \bar{\phi}^2 \ln(l)$$

Mystery 2: Unstable or Metastable?



When *slightly* perturbed, does a sf vortex
fall apart immediately, or
remain intact?

Numerical analysis of stability: Method

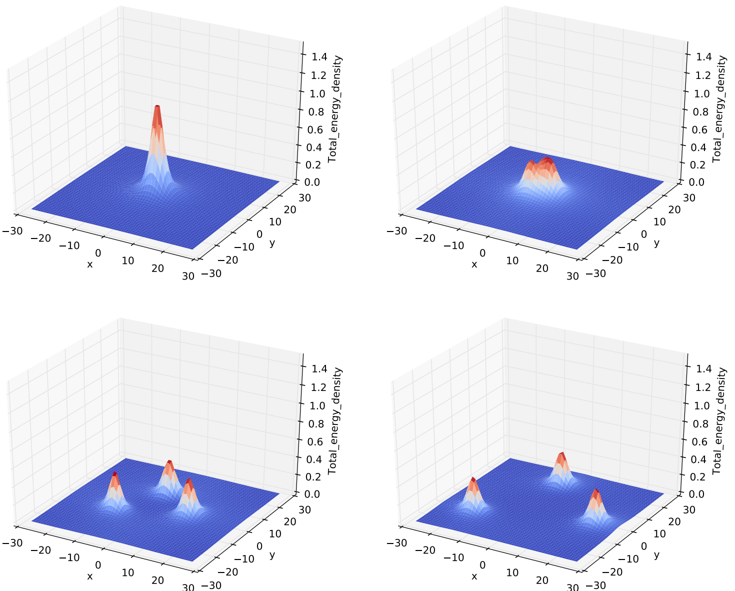
- Discretize scalar and gauge fields on a 2D lattice
- Choose couplings in the effective theory

$$U(\Phi) = m^2 \mathrm{Tr}[\Phi^\dagger \Phi] + \lambda_1 (\mathrm{Tr}[\Phi^\dagger \Phi])^2 + \lambda_2 \mathrm{Tr}[(\Phi^\dagger \Phi)^2]$$

  gauge coupling g
  condensate self-couplings λ_1, λ_2 ($\lambda \equiv 3\lambda_1 + \lambda_2$)
- Initial config: superfluid vortex plus a small random perturbation
- Evolve forward in time and see what happens:
  Unstable: an unstable mode grows exponentially until the vortex falls apart
  Metastable: the vortex experiences oscillations that do not grow in amplitude.
- Vary the couplings, and map out *metastability boundary* in space of couplings

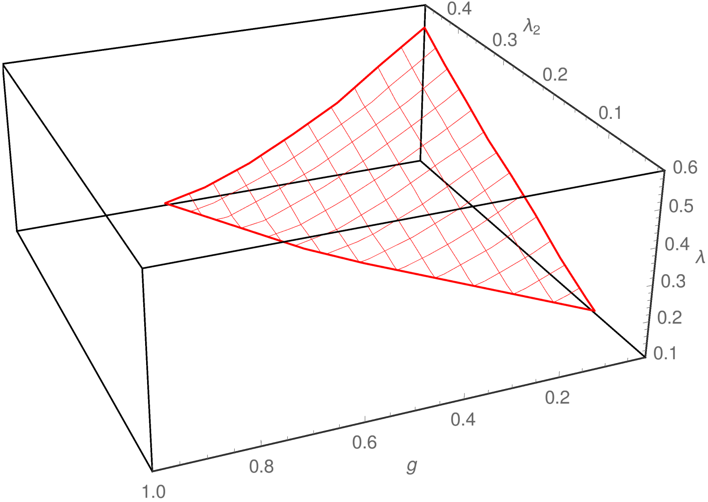
Numerical analysis of stability: example

Energy density plot, showing decay of a sf vortex



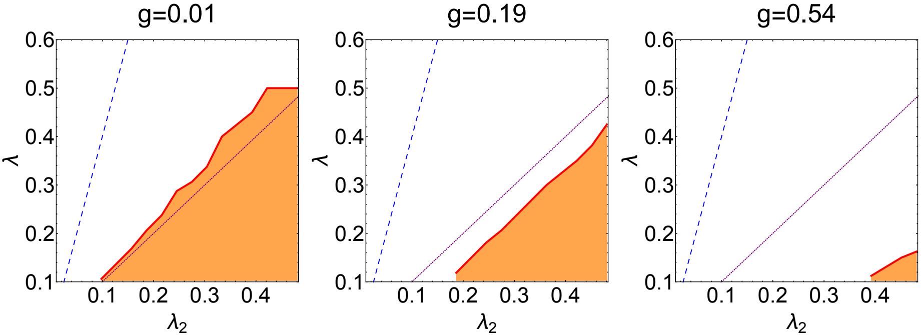
Numerical analysis of stability: Results

Metastability region: $\lambda_1 = \frac{1}{3}(\lambda - \lambda_2)$



Vortices are metastable at low g and sufficiently negative λ_1;
varying λ_2 at fixed λ_1 does not make much difference

Numerical analysis of stability: Results



Superfluid vortices are metastable when $\lambda_1 \lesssim -0.16g$ $\quad (\lambda_1 = \frac{1}{3}(\lambda - \lambda_2))$

Increasing g or λ_1 drives instability

Increasing λ_2 at fixed g and λ_1 doesn't make much difference.

Can we understand the role of λ_1?

What mode initiates vortex decay?

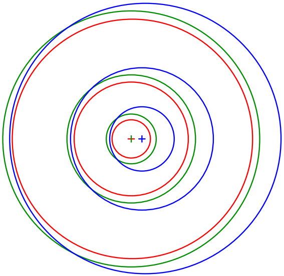
At $g = 0$ (no color gauge fields) we can guess the unstable mode analytically.

superfluid vortex:

$$\Phi^{(\mathrm{sf})a}_{\ \ \alpha} = \begin{array}{c} \\ \color{red}{\mathrm{r}} \\ \color{green}{\mathrm{g}} \\ \color{blue}{\mathrm{b}} \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{u} & \mathrm{d} & \mathrm{s} \end{array} \\ \begin{pmatrix} \varphi(\vec{r}) & & \\ & \varphi(\vec{r}) & \\ & & \varphi(\vec{r}) \end{pmatrix} \end{array} \qquad \varphi(\vec{r}) \equiv \bar{\phi}\, e^{i\theta}\, \beta(r)$$

Now, suppose we shift the different color/flavor components apart. Shift red and green to the left by ϵ, and blue to the right by 2ϵ

$$\Phi^{(\mathrm{sf})\ a}_{\mathrm{pert}\ \alpha} = \begin{array}{c} \\ \color{red}{\mathrm{r}} \\ \color{green}{\mathrm{g}} \\ \color{blue}{\mathrm{b}} \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{u} & \mathrm{d} & \mathrm{s} \end{array} \\ \begin{pmatrix} \varphi(\vec{r}+\epsilon\hat{x}) & & \\ & \varphi(\vec{r}+\epsilon\hat{x}) & \\ & & \varphi(\vec{r}-2\epsilon\hat{x}) \end{pmatrix} \end{array}$$



The unstable mode of a vortex

So the perturbation is

$$\delta\Phi^{a}_{\alpha} = \epsilon\, \hat{x} \cdot \vec{\nabla}\varphi(\vec{r})\, T_8{}^{a}_{\alpha} \qquad T_8 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Calculating how this changes the energy, we find

$$\delta E = -\epsilon^2 \lambda_1 \frac{3\pi m^4}{(\lambda_2 + 3\lambda_1)^2} \int_0^\infty \Big(\frac{d\beta}{dr}\Big)^2 \beta^2 r dr$$

If λ_1 is positive, this lowers the energy: vortex is unstable.

In the numerical evolution, $\delta\Phi$ matches the mode that is observed to grow exponentially fast in the Unstable region of parameter space.

We appear to have guessed the unstable mode at small g!

Summary

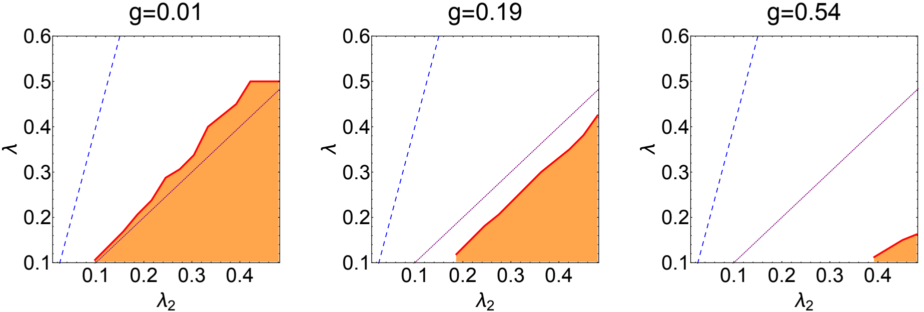
- The CFL phase of quark matter is a superfluid and so should carry angular momentum in $n=1$ vortices. However, the vortex has higher energy than three well-separated $n=\frac{1}{3}$ semi-sf flux tubes.
- Semi-sf flux tubes have lower energy because their color flux partly cancels the gradient energy ($E \sim n^2$).
- Depending on the couplings in the effective theory, a vortex may be metastable or unstable against decay.
- Weak coupling QCD calculations say that they are unstable.
- The mode that initiates decay does not involve the gauge fields!
- Semi-sf flux tubes are the only known example of long-range color gauge potentials

Further questions

- **Quark-hadron continuity** (Schäfer & Wilczek hep-ph/9811473)
  hyperonic matter: superfluid with global vortices
  CFL quark matter: superfluid with semi-superfluid flux tubes
  Do the long-range color fields of a ssf flux tube provide a way to distinguish CFL from hyperonic matter?
  Alford, Baym, Fukushima, Hatsuda arXiv:1803.05115
  Cherman,Sen,Yaffe, arXiv:1808.04827
- We assumed perfect flavor symmetry. Need to include strange quark mass and electric neutrality constraint.
- Include entrainment (current-current) interactions?
- Stability of vortices in "color-spin-locked" phase of quark matter
  (Schäfer, hep-ph/0006034)
- Observable consequences for stars with CFL cores?
  - semi-sf flux tubes pin to LOFF crystal differently from sf vortices?
  - zero modes of flux tubes play a role in transport?

Additional slides

Real world CFL matter



The couplings in the effective theory are determined by microscopic physics.

Weak coupling calculation: $\lambda_1 = \lambda_2 \approx 420\left(\frac{T_c}{\mu_q}\right)^2$

E.g. $T_c = 15\,\text{MeV},\ \mu_q = 400\,\text{MeV} \Rightarrow \lambda_2 \approx 0.6$

(Iida, Baym, hep-ph/0011229; Giannakis, Ren, hep-ph/0108256)

This gives the dashed line in the figure

If this calculation can be extrapolated down to neutron star densities, CFL vortices would *always* be unstable.